EXERCISE – II — HINTS & SOLUTIONS

Sol.1 A,B,C,D

sinx, sin2x, sin 3x in A.P.

2sin2x = sin x + sin 3x

$\Rightarrow$ 2sin 2 x = 2 sin 2x cos x $\Rightarrow$ sin 2x (cox – 1) = 0

$\Rightarrow$ sin 2 x = 0 or cos x = 1

2x = nπ or x = 2nπ n $\in$ I

$$x = \frac{n\pi}{2},\ n \in I$$

If n = 2m $\Rightarrow$ x = mπ, m $\in$ I

If m is odd $\Rightarrow$ x = (2k + 1)π, k $\in$ I

Sol.2 A,D

sin x – cos^{2}x – 1

sin^{2}x + sinx – 2 min value of quadratic expression is

$$\frac{-D}{4a} \text{ at } \frac{-b}{2a} \Rightarrow \frac{-D}{4a} = -\frac{1+8}{4} = -\frac{9}{4} \text{ at } \frac{-b}{2a} = -\frac{1}{2}$$

$\Rightarrow$ Given expression has min. value at sinx = $-\frac{1}{2}$

$$\Rightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) \Rightarrow x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right), n \in I$$

$$\Rightarrow x = n\pi + (-1)^{n+1}\frac{\pi}{6} \text{ or } n\pi - (-1)^n\frac{\pi}{6},\ n \in I$$

Sol.3 B,D

sinx + sin 2x + sin 3x = 0

$$\Rightarrow \sin 2x + 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{3x-x}{2}\right) = 0$$

$\Rightarrow$ sin 2x + 2 sin 2x cos x = 0

$\Rightarrow$ sin 2x [1 + 2 cos x] = 0

$\Rightarrow$ sin 2x = 0 or cos x = $-\frac{1}{2}$

Sol.4 B,C

cos 4x . cos 8x – cos 5x . cos 9x = 0

$\Rightarrow$ 2 cos 4x cos 8x – 2 cos 5x cos 9x = 0

$\Rightarrow$ cos 12x + cos 4x – cos 14 x – cos 4x = 0

$\Rightarrow$ cos 12 x = cos 14x

$\Rightarrow$ cos 12x – cos 14 x = 0 $\Rightarrow$ 2 sinx sin 13x = 0

$\Rightarrow$ sin x = 0 or sin 13 x = 0

Sol.5 A,C

cosx cos 6x = – 1

$\Rightarrow$ cosx 7x + cos 5x = –2

If's only possible that

cos 7x = –1 & cos 5x = –1

7x = (2n + 1) π & 5x = (2m + 1)π, n, m $\in$ I

$$x = (2n+1)\frac{\pi}{7} \quad \& \quad x = (2m+1)\frac{\pi}{5}$$

$$x = \frac{\pi}{7}, \frac{3\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}$$

$$\& \quad x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$$

common solution in one round is π

In general x = (2n ± 1) π ,n $\in$ I

Sol.6 B,C,D

If sin (x – y) = cos (x + y) = $\frac{1}{2}$ 0 < x, y < π

sin (x – y) = $\frac{1}{2}$ & cos (x + y) = $\frac{1}{2}$

$$x - y = \frac{\pi}{6}, \frac{5\pi}{6},\ (x+y) = \frac{\pi}{3}, \left(2\pi - \frac{\pi}{3}\right)$$

$$x + y = \frac{\pi}{3}, \frac{5\pi}{3}$$

{add small & subtraction is large not possible}

$$x - y \neq \frac{5\pi}{6} \ \& \ x + y \neq \frac{\pi}{3}$$

So x – y = $\frac{\pi}{6}$ or x – y = $\frac{\pi}{6}$ or x – y = $\frac{5\pi}{6}$

& x + y = $\frac{\pi}{3}$ x + y = $\frac{5\pi}{3}$ x +y = $\frac{5\pi}{3}$

$x = \frac{\pi}{5}, y = \frac{\pi}{12}$ $x = \frac{11\pi}{12}, y = \frac{3\pi}{4}$ $x = \frac{5\pi}{4}, y = \frac{5\pi}{12}$

Sol.7 A,B,C,D

$$2\sin\frac{x}{2}.\cos^2x + \sin^2x = 2\sin\frac{x}{2}.\sin^2x + \cos^2x = 0$$

$$\Rightarrow 2\sin\frac{x}{2}(\cos^2x - \sin^2x) - (\cos^2 - \sin^2x) = 0$$

$$\Rightarrow (\cos^2x - \sin^2x)\left(2\sin\frac{x}{2} - 1\right) = 0$$

$\Rightarrow \cos 2x \left(2 \sin \frac{x}{2} - 1\right) = 0$

$\Rightarrow \cos 2x = 0$ or $\sin \frac{x}{2} = \frac{1}{2}$

$\Rightarrow \sin 2x = \pm 1$ or $\cos^2 \frac{x}{2} = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \cos x = 2 \left(\frac{3}{4}\right) - 1 = \frac{1}{2}$

$\Rightarrow \cos 2x = 2 \left(\frac{1}{4}\right) - 1 = -\frac{1}{2}$

Sol.8 A,B,C,D

$\cos 15x = \sin 5x$ if

$\cos 15x = \cos \left(\frac{\pi}{2} - 5x\right)$

$15x = 2n\pi \pm \left(\frac{\pi}{2} - 5x\right)$

$15x = 2n\pi + \frac{\pi}{2} - 5x$ or $15x = 2n\pi - \frac{\pi}{2} + 5x$

$20x = 2n\pi + \frac{\pi}{2}, n \in I$ or $10x = 2n\pi - \frac{\pi}{2}, n \in I$

$x = \frac{n\pi}{10} + \frac{\pi}{40}, n \in I$ or $x = \frac{n\pi}{5} - \frac{\pi}{20}, n \in I$

Put $n = n - 1$ put $n = n + 1$

$x = \frac{n\pi}{10} - \frac{\pi}{10} + \frac{\pi}{40}$ $x = \frac{n\pi}{5} + \frac{\pi}{5} - \frac{\pi}{20}$

$x = \frac{n\pi}{10} - \frac{3\pi}{40}, n \in I$ $x = \frac{n\pi}{5} + \frac{3\pi}{20}, n \in I$

Sol.9 A,C

$5 \sin^2 x + \sqrt{3} \sin x \cos x + 6\cos^2 x = 5(1)^2$

$\Rightarrow 5 \sin^2 x + \sqrt{3} \sin x \cos x + 6 \cos^2 x$

$= 5 \sin^2 x + 5 \cos^2 x$

$\Rightarrow \cos^2 x + \sqrt{3} \sin x \cos x = 0$

$\Rightarrow \cos x [\cos x + \sqrt{3} \sin x] = 0$

$\Rightarrow \cos x \left[\frac{1}{2}.\cos x + \frac{\sqrt{3}}{2}\sin x\right] = 0$

$\Rightarrow \cos x \left[\cos\left(x - \frac{\pi}{3}\right)\right] = 0$

$\Rightarrow \cos x = 0$ or $\cos \left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow x = (2n + 1) \frac{\pi}{2}$ or $x - \frac{\pi}{3} = 2m\pi \pm \frac{\pi}{2}; m \in I$

$\Rightarrow x = n\pi + \frac{\pi}{2}, n \in I$ or $x = 2m\pi + \frac{\pi}{2} + \frac{\pi}{3}; m \in I$

$\Rightarrow x = 2m\pi - \frac{\pi}{2} + \frac{\pi}{3}$ or $x = 2m\pi - \frac{\pi}{6}$

$\tan x = -\frac{1}{\sqrt{3}}$

Sol.10 C,D

$\sin^2 x + 2 \sin x \cos x - 3\cos^2 x = 0$

$\Rightarrow \sin^2 x + 3 \sin x \cos x - \sin x \cos x - 3 \cos^2 x = 0$

$\Rightarrow (\sin x + 3 \cos x) (\sin x - \cos x) = 0$

$\Rightarrow \sin x + 3 \cos x = 0$ or $\sin x - \cos x = 0$

$\because \sin x \neq 0$ & $\cos x \neq 0$

$\Rightarrow \tan x = -3$ or $\tan x = 1$

$x = n\pi + \tan^{-1}(-3)$ or $x = n\pi + \frac{\pi}{4}; n \in I$

Sol.11 B,C

$\sin^2 x - \cos 2x = 2 - \sin 2x$

$\Rightarrow \sin^2 x - 2\cos^2 x + 1 = 2 - \sin 2x$

$\Rightarrow 1 - \cos^2 x - 2 \cos^2 x + 1 = 2 - \sin 2x$

$\Rightarrow \sin 2x - 3\cos^2 x = 0$

$\Rightarrow \cos x [2\sin x - 3 \cos x] = 0$

$\Rightarrow \cos x = 0$ or $2 \sin x - 3 \cos x = 0$

$\Rightarrow x = (2n + 1) \frac{\pi}{2}, n \in I$ or $2 \sin x = 3 \cos x$

$\tan x = \frac{3}{2}$